St Wilfrid's Catholic Primary School

Manipulatives and Written Methods of Calculation

Progression of strategies from Year 1 to Year 6

Examples of questions to help support pupil's understanding

Starting a piece of work:

How are you going to tackle this?
What information do you have?
What do you need to find out or do?
Which operation is applicable?
Is it a mental question? Do you need to jot something down?
What equipment might help you?
What questions might you ask?
How are you going to record what you are doing?
What do you think the result might be? Can you estimate or predict?

When a pupil has become 'stuck':

Can you describe the problem in your own words?
Can you talk me through what you have done so far?
What did you do last time?
Is it different this time?
Is there something you already know which might help you?
Could you try it with simpler numbers, a number line, less numbers?
Would a table, picture, diagram or graph help you?
Make a guess (gut reaction) and see if it works.

While working:

Can you explain what you have done so far?
What else is there to do?
Why did you decide this method?
Is it working? Is it effective?
Is there a quicker way of doing this?
What do you mean by?
What did you notice when?
Why did you organise your results like that?
Is a pattern or rule emerging?
Do you think this will work with other numbers?
Have you thought of all possibilities? Are you sure?

When a pupil has finished:

How did you get your answer?
Can you describe your method, pattern, rule to us?
Can you explain why it works?
What might you try next?
What if you started with…rather than?
Is it a reasonable result?
Explain why. How did you check it?
What have you found out today?
If you did it again, what would you do differently?
Having done this, when might you use this information again?
Did you use any new words today? What do they mean?

Foundation

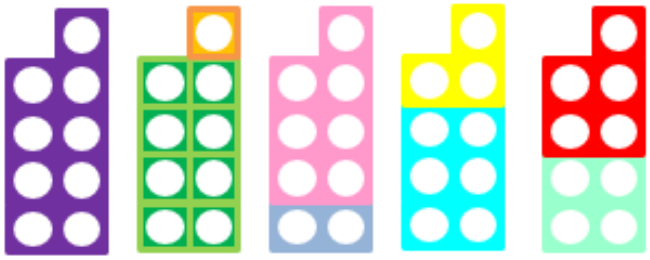
Stages in counting

All children go through these stages in counting. Generally, they should be secure with them by the end of Year R.

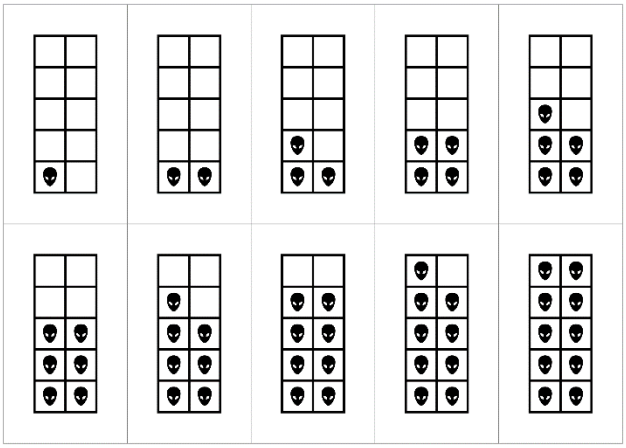
1. Stable order (knowing numbers come in an order)
2. One to one correspondence (touching and counting)
3. Cardinal (knowing last number is the total)
4. Order irrelevance (doesn't matter how you count the total will be the same)
5. Abstraction (being able to count without seeing/touching items)

By the end of Year R children should also be able to:

- Subitise (know number of dots on dice or dominoes without counting)
- Know about the numbers to 10, for example, 9 is made up of 1 and 8, 2 and 7 etc., it is greater than 4, less than 10



- Recognise and begin to write numerals



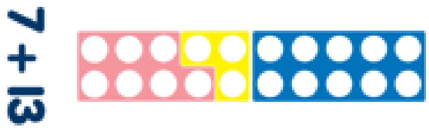
Progression towards the written methods

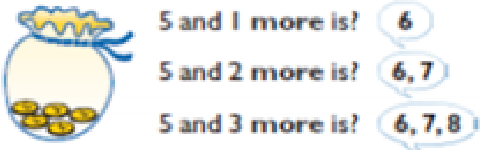
Use counters, bead strings, any 'home objects' and progress to Numicon to encourage counting on from one number to find the sum of quantities to 10 and, if appropriate, to 20.

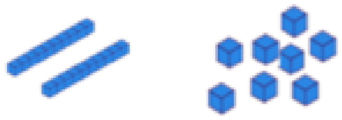
Year 1

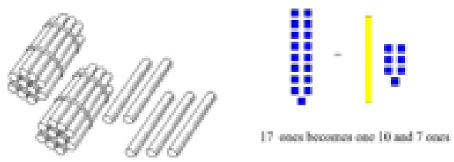
Addition

Add and subtract 1 digit and 2 digit numbers to 20 including zero.



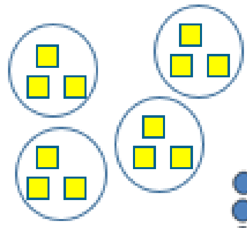




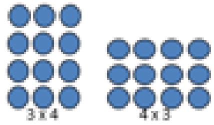


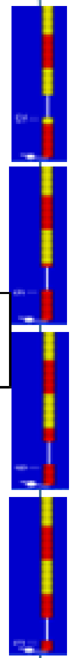
Multiplication

Count in multiples of 2, 5 and 10



4 groups of 3 = 3 four times = 3 x 4

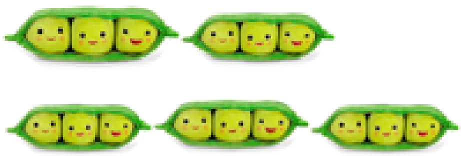




Set counters out as arrays and explore commutativity and early inverse.

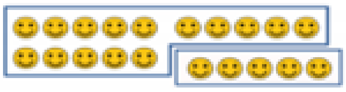
There are 3 peas in a pod. How many peas are in 5 pods altogether?

3 + 3 + 3 + 3 + 3 = 15



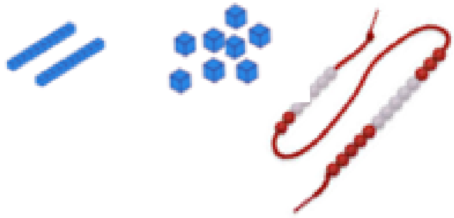
Subtraction

20 – 5 = 15 15 + 5 etc



Find the difference to 20 or to 50 if appropriate.

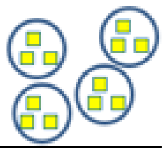
Write a number statement/draw a picture to show what you have done.



Division

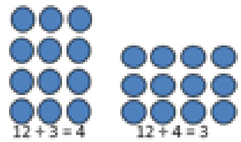
12 ÷ 3

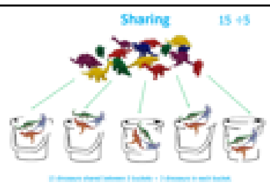
How many groups of 3 in 12?



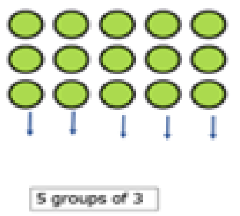
Bead strings

Arrays

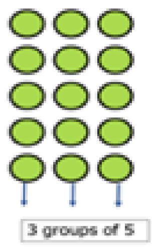




Grouping



Arrays

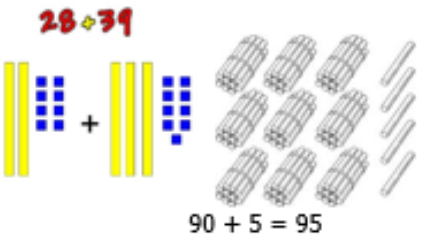


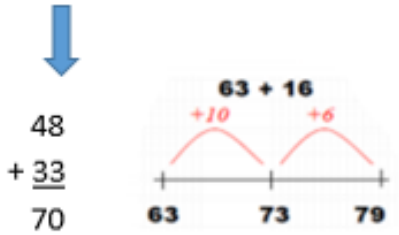
How many 3s in 15?

Year 2

Add and subtract two-digit number and ones , a two-digit number and tens , two, two-digit numbers, adding three one-digit numbers up to 100.

Addition





48
+ 33
70
11
81

Summer term Introduce vertical partitioning.

Multiplication

4 x 5 = ___

4 x ? = 20

5 x 4 = etc

Use arrays:

Fingers

Subtraction

Include exchange with manipulatives.

Year 2: Continue using Dienes to find differences between quantities to 100, Writing number statements/drawing pictures to show what they have done.

65 – 47

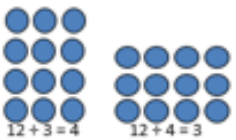
Take away 40

Exchange one 10 for 10 ones in order to take away 7

Division

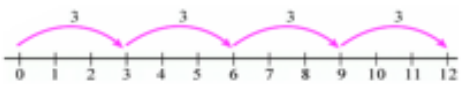
How many groups of 4 can be made with 12 stars? = 3

Arrays



Fingers

Grouping Number Lines



A CD costs £3. How many CDs can I buy with £12?

Year 3

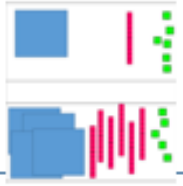
Add and subtract numbers with up to three digits, using formal written methods of column

Addition

3 digit examples that cannot be answered using a mental method. Manipulatives must be used to explore why we begin with the least significant number.

```
 248          leading to      248
+139                         +139
 300                          387
  70                           1
  17
 387
```

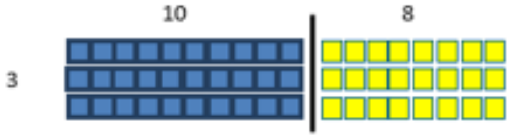
What is the same? What is different?



Calculate statements using known tables including 2 digits x 1 digit. Progressing to formal methods. Do not use multipliers which lend themselves to a mental method (2,4,5,10)

Multiplication

Arrays to support the grid method



	10	8
3	30	24

18 x 3 = 54

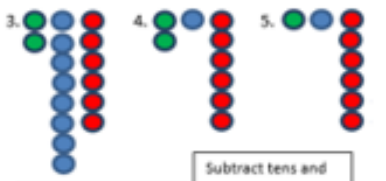
leading to written method

```
 252
 x 6
1512
 3 1
```

Use of manipulatives to lead to compact method:

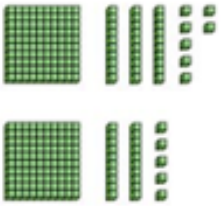
Subtraction

```
 2 8 9 14
 - 1 7 8
 1 1 6
```



Exchange one ten for ones and subtract 8

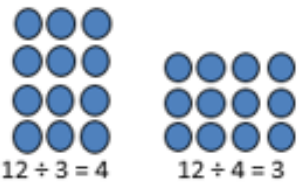
Subtract tens and then hundreds



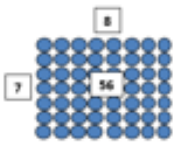
Division

Use arrays and then move towards formal written methods using manipulatives. How many groups of ...

Arrays

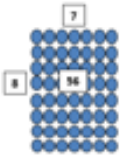


12 ÷ 3 = 4 12 ÷ 4 = 3



Introducing the conceptual variation of the written method with division bracket.

$$7\overline{)56}$$

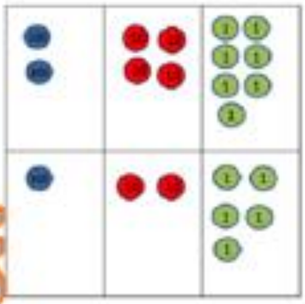


$$8\overline{)56}$$

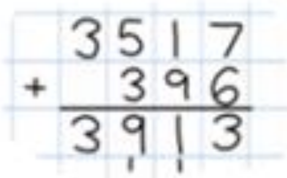
Year 4

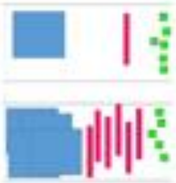
Add and subtract numbers with up to 4 digits using the formal written methods of columns with manipulatives to show exchange

Addition



```
 247
+125
  12
  60
 300
 372
```

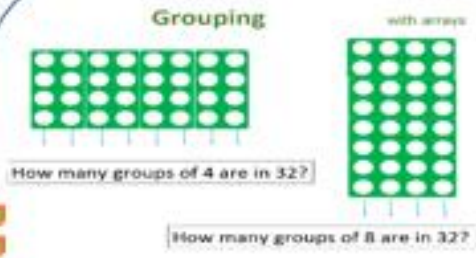


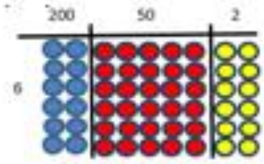


```
 2634
+4517
 7151
```

Multiplication

Multiply 2 digit and 3 digit numbers by a one digit using formal method.





	200	50	2
6	1200	300	12

= 1512

Or...

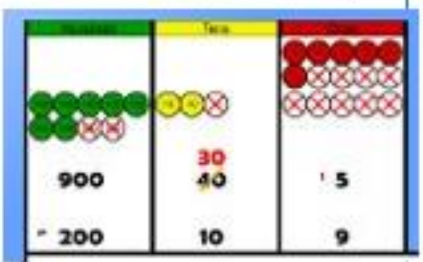
```
 252
 x 6
  12
 300
1200
1512
```

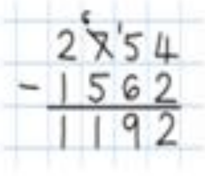
leading to written method

```
 252
  x6
1512
```

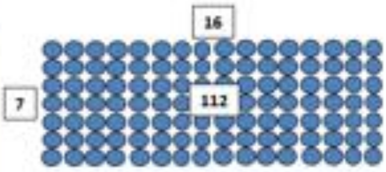
Subtraction

Use of manipulatives to lead to compact method:

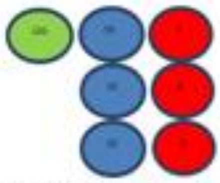




Division



From this we know:
7 x 16 = 112
16 x 7 = 112
112 ÷ 16 = 7
112 ÷ 7 = 16



You cannot take 6 groups of 100 away from the one 100

Exchange the 100 for 10 tens so you have 13 tens

6 | 1¹3 3

You can now take two groups of 6 tens

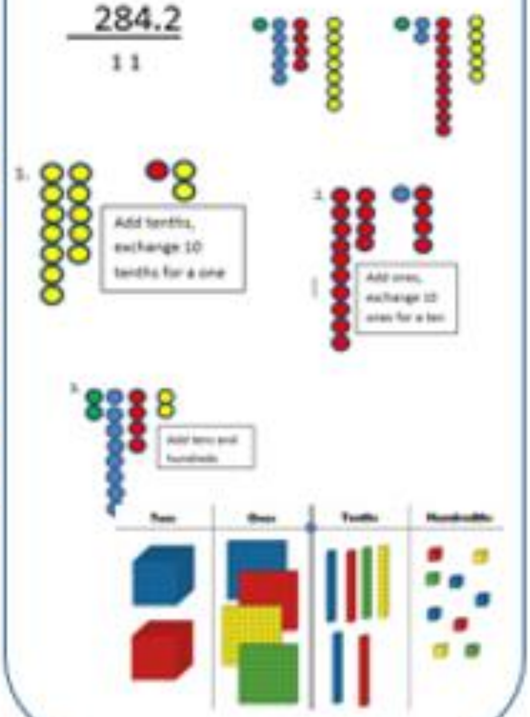
YEAR 5 Add and subtract whole numbers with more than 4 digits using formal written methods

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Addition

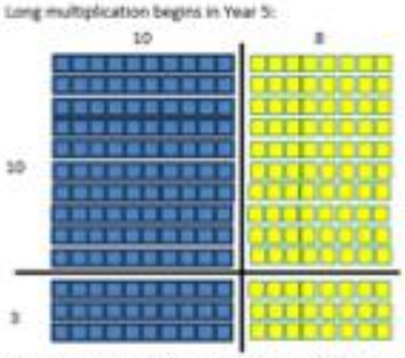
Written method: decimals (with manipulatives first)

```
  154.7
+ 129.5
  284.2
  1 1
```



Multiplication

Long multiplication begins in Year 5:



Also use Dienes and place value counters to demonstrate this

	10	8	
10	100	80	180
3	30	24	54
		Total:	234

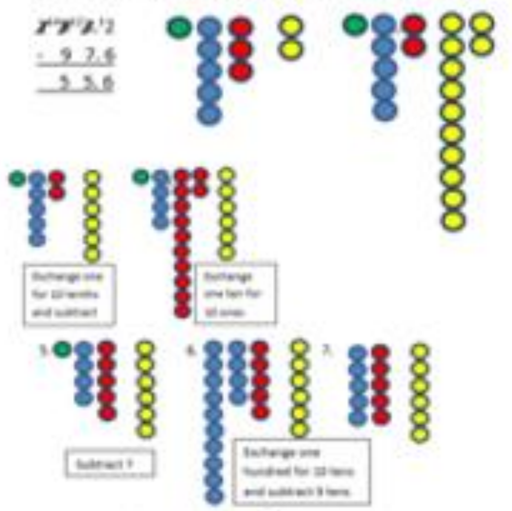
Leading to written method:

```
  18
x 13
  54
 180
 234
```

```
Or…
  18     18     180
x 10    x 3   + 54
 180     54    234
```

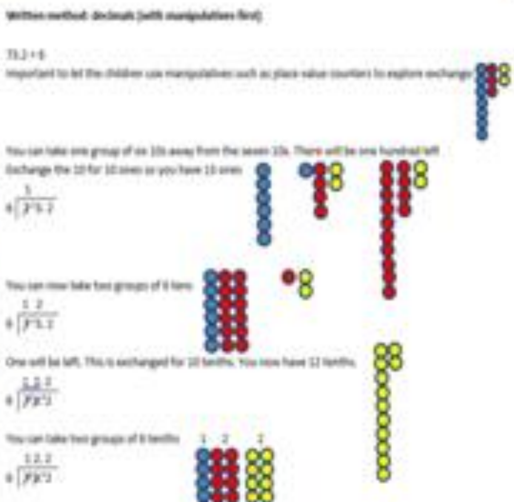
Subtraction

Written method: decimals (with manipulatives first)



```
  ⁶7̸ 1̸ ¹6 9̸ · ¹0
−     3  7  2 · 5
   6  7  9  6 · 5
```

Division



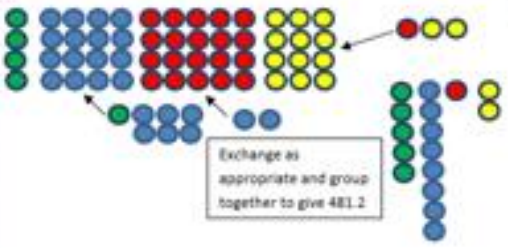
Year 6

Addition

	2	3	·	3	6	1
		9	·	0	8	0
	5	9	·	7	7	0
+		1	·	3	0	0
	9	3	·	5	1	1
	2	1		2		

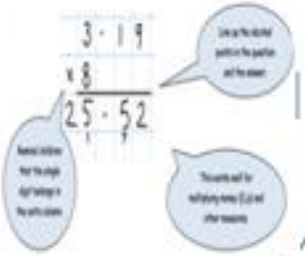
	8	1	,	0	5	9
		3	,	6	6	8
	1	5	,	3	0	1
+	2	0	,	5	5	1
	1	2	0	,	5	7	9
		1	1	1	1	

Multiplication



	4	5	6	2
×			7	2
	9	1	2	4
3	1	9	3	4	0
3	2	8	4	6	4

145.3
× 4
581.2



Subtraction

	~~8~~	~~10~~	,	6	9	9
-		8	9	4	9	
		6	0	,	7	5	0

	~~1~~	~~0~~	5	·	~~4~~	1 9 kg
-		3	6	·	0	8 0 kg
		6	9	·	3	3 9 kg

Empty decimal places can be filled with **zero** to show the place value in each column.

Division

Short Division

with remainders to 3 decimal places

T	H	T	U.	tenths	hundredths	thousandths
0	8	1	2.	1	2	5

8) 6 4 9 7.0 0 0

(6) (1) (1) (2) (4)

What do you think is the equivalent % and fraction of this remainder?

432 ÷ 15 becomes

		2	8	
1	5	4	3	2
		3	0	0	15×20
		1	3	2	
		1	2	0	15×8
			1	2	

$\frac{12}{25} = \frac{4}{5}$

432 ÷ 15 becomes

		2	8 ·	8
1	5	4	3	2 · 0
		3	0	↓
		1	3	2
		1	2	0 ↓
			1	2 0
			1	2 0
				0